

# Big-bang nucleosynthesis with a long-lived charged massive particle including ${}^4\text{He}$ spallation processes

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We propose helium-4 spallation processes induced by long-lived stau in supersymmetric standard models, and investigate an impact of the processes on light elements abundances. We show that, as long as the phase space of helium-4 spallation processes is open, they are more important than stau-catalyzed fusion and hence constrain the stau property.

## I. INTRODUCTION

Quests for the physics beyond the Standard Model (SM) will reach a new stage at the TeV scale. Among the expected interesting signals of the new physics are those provided by exotic charged particles (charged massive particles; CHAMPs) with a long lifetime. The presence of such particles is predicted in many notable models beyond the SM, although its identity depends on the models one assumes. CHAMP hunting is indeed one of the major issues of the high energy experiments, and its collider phenomenology is enthusiastically studied [1–14]; it also motivates other researches including neutrino telescope observations [15–17] and cosmology [18–21].

Long-lived CHAMPs will play interesting roles in the Big-Bang Nucleosynthesis (BBN) as well. The light nuclei will interact not only with the CHAMPs during the BBN processes [22–39], but also with the decay products of the CHAMPs in the post-BBN era [40–47]. The standard scenario of the BBN will thus be altered, and so is the abundance of the light elements at the present time. One can thus constrain the models beyond the Standard Model by evaluating their prediction on the light elements abundance and comparing it with the current observations. We can then give stringent predictions for the forthcoming experiments and observations according to these constraints.

The Standard Model extended with supersymmetry (SUSY) is one of the models that can accommodate such long-lived CHAMPs. With the  $R$ -parity conservation, the lightest SUSY particle (LSP) is stable and become a cold dark matter. Interestingly, it can offer a long-lived CHAMPs if the LSP is the bino-like neutralino  $\tilde{\chi}_1^0$ . Coannihilation mechanism is required to account for the dark matter abundance in this case [48], where the LSP and the next-lightest SUSY particle (NLSP) are almost degenerate in mass. Staus, denoted by  $\tilde{\tau}$  and a possible

candidate of the NLSP, can acquire a long lifetime when the mass difference with the LSP is less than the mass of tau leptons. This is due to the phase space suppression of the final state that necessarily consists of three particles or more. Noting that such long-lived staus will be copious during the BBN [49, 50], we have shown in [28, 32, 37] that their presence indeed alters the prediction of the standard BBN and possibly solve the discrepancy of the lithium abundance in the Universe through the internal conversion reactions.

In this article, we improve our analyses by including new reactions of

$$(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + t + n, \quad (1a)$$

$$(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + d + n + n, \quad (1b)$$

$$(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + p + n + n + n, \quad (1c)$$

in which  $(\tilde{\tau} {}^4\text{He})$  represents a bound state of a stau and  ${}^4\text{He}$  nucleus. Reaction (1) is essentially a spallation of the  ${}^4\text{He}$  nucleus, producing a triton  $t$ , a deuteron  $d$ , and neutrons  $n$ . Presence of such spallation processes has been ignored so far due to the naïve expectation that the rate of the stau-catalyzed fusion [22]

$$(\tilde{\tau} {}^4\text{He}) + d \rightarrow \tilde{\tau} + {}^6\text{Li} \quad (2)$$

is larger than the reaction (1). Indeed, the cross section of Eq. (2) is much larger than that of  ${}^4\text{He} + d \rightarrow {}^6\text{Li} + \gamma$  by  $(6 - 7)$  orders of magnitude [51].

We point out that this expectation is indeed naïve; the reaction Eq. (1) is more effective than Eq. (2) as long as the spallation processes are kinematically allowed. The former reaction rapidly occurs due to the large overlap of their wave functions in a bound state. On the other hand, the latter proceeds slowly since it requires an external deuteron which is sparse at the BBN era. The overproduction of  $t$  and  $d$  is more problematic than that of  ${}^6\text{Li}$ . This puts new constraints on the parameters of

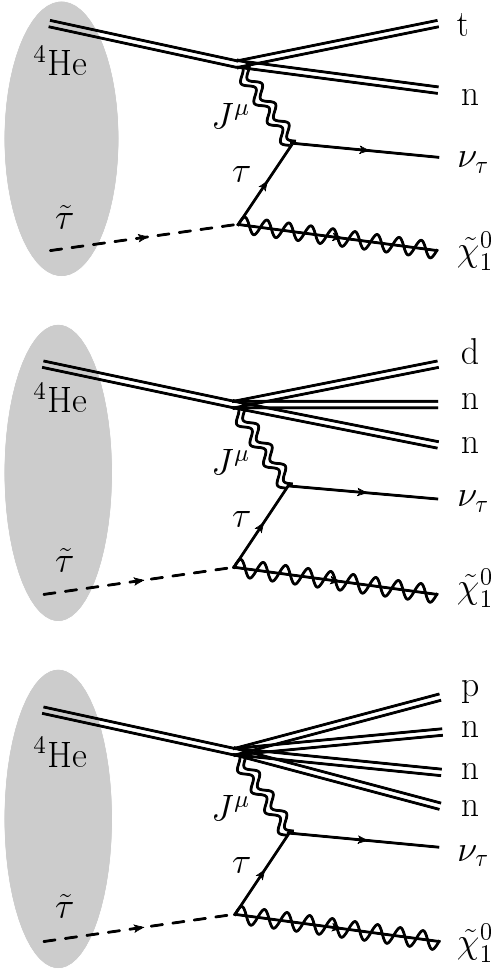


FIG. 1:  ${}^4\text{He}$  spallation processes.

the minimal supersymmetric standard model (MSSM). Note that there is no reaction corresponding to Eq. (1) in the gravitino LSP scenario [52].

The purpose of this work is to understand the impact of  ${}^4\text{He}$  spallation processes on light element abundances. In Section II, we analytically calculate its reaction rates, and compare its timescale with that of the reaction Eq. (2). In Section III, we calculate all of light element abundances including exotic reactions, i.e., the  ${}^4\text{He}$  spallation processes, the stau-catalyzed fusion, and the internal conversion processes. We show the MSSM parameter space in which we can reproduce the observed abundances of both dark matter and light elements including  ${}^7\text{Li}$  and  ${}^6\text{Li}$ . Section IV is devoted to a summary.

## II. SPALLATION OF HELIUM 4

Two types of reactions are possible for the bound state of a stau and a  ${}^4\text{He}$  nucleus: (1) the stau-catalyzed fusion and (2) the spallation of the  ${}^4\text{He}$  nucleus. The property of stau is stringently constrained in order to evade the

overproduction of the various light elements due to these processes.

In this section, we calculate the rate of the spallation of the  ${}^4\text{He}$  nucleus. We compare the result with the rate of the stau-catalyzed fusion to show that the former is larger than the latter in a large part of the parameter space, and thereby show that the spallation plays a significant role in the BBN.

The  ${}^4\text{He}$  spallation processes of Eq. (1) is described by the Lagrangian

$$\mathcal{L} = \tilde{\tau}^* \tilde{\chi}_1^0 (g_L P_L + g_R P_R) \tau + \sqrt{2} G_F \nu_\tau \gamma^\mu P_L \tau J_\mu + \text{h.c.}, \quad (3)$$

where  $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$  is the Fermi coupling constant,  $P_{L(R)}$  represents the chiral projection operator, and  $J_\mu$  is the weak current. The effective coupling constants  $g_L$  and  $g_R$  are given by

$$g_L = \frac{g}{\sqrt{2} \cos \theta_W} \sin \theta_W \cos \theta_\tau, \quad (4)$$

$$g_R = \frac{\sqrt{2} g}{\cos \theta_W} \sin \theta_W \sin \theta_\tau e^{i\gamma_\tau},$$

where  $g$  is the  $SU(2)_L$  gauge coupling constant and  $\theta_W$  is the Weinberg angle. The mass eigenstate of staus is given by the linear combination of  $\tilde{\tau}_L$  and  $\tilde{\tau}_R$ , the superpartners of left-handed and right-handed tau leptons, as

$$\tilde{\tau} = \cos \theta_\tau \tilde{\tau}_L + \sin \theta_\tau e^{-i\gamma_\tau} \tilde{\tau}_R. \quad (5)$$

Here  $\theta_\tau$  is the left-right mixing angle of staus and  $\gamma_\tau$  is the CP violating phase.

### A. $(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + \mathbf{t} + \mathbf{n}$

First we consider the process of Eq. (1a). The rate of this process is expressed as

$$\frac{1}{\tau_{\text{tn}}} = \frac{1}{|\psi|^2 \cdot \sigma v_{\text{tn}}}, \quad (6)$$

where  $|\psi|^2$  stands for the overlap of the wave functions of the stau and the  ${}^4\text{He}$  nucleus. We estimate the overlap by

$$|\psi|^2 = \frac{(Z\alpha m_{\text{He}})^3}{\pi}, \quad (7)$$

where  $Z$  and  $m_{\text{He}}$  represent the atomic number and the mass of  ${}^4\text{He}$ , respectively, and  $\alpha$  is the fine structure constant. We assumed that the stau is pointlike particle and is much heavier than  ${}^4\text{He}$  nucleus so that the reduced mass of the bound state is equal to the mass of  ${}^4\text{He}$  nucleus itself. The cross section of the elementary process

for this reaction is denoted by  $\sigma v_{\text{tn}}$  and calculated as

$$\begin{aligned} \sigma v_{\text{tn}} &\equiv \sigma v((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{tn}) \\ &= \frac{1}{2E_{\tilde{\tau}}} \int \frac{d^3 \mathbf{p}_\nu}{(2\pi)^3 2E_\nu} \frac{d^3 \mathbf{p}_{\tilde{\chi}}}{(2\pi)^3 2E_{\tilde{\chi}}} \frac{d^3 \mathbf{q}_n}{(2\pi)^3} \frac{d^3 \mathbf{q}_t}{(2\pi)^3} \\ &\quad \times |\mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{tn})|^2 \\ &\quad \times (2\pi)^4 \delta^{(4)}(p_{\tilde{\tau}} + p_{\text{He}} - p_\nu - q_t - q_n). \end{aligned} \quad (8)$$

Here  $p_i$  and  $E_i$  are the momentum and the energy of the particle species  $i$ , respectively.

We briefly show the calculation of the amplitude of this process, leaving the full calculation in Appendix. The amplitude is deconstructed as

$$\begin{aligned} \mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{tn}) &= \langle \text{tn} | \tilde{\chi}_1^0 \nu_\tau | \mathcal{L}_{\text{int}} | {}^4\text{He} \tilde{\tau} \rangle \\ &= \langle \text{tn} | J^\mu | {}^4\text{He} \rangle \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle. \end{aligned} \quad (9)$$

Here we omitted the delta function for the momentum conservation and the spatial integral. The weak current  $J_\mu$  consists of a vector current  $V_\mu$  and an axial vector current  $A_\mu$  as  $J_\mu = V_\mu + g_A A_\mu$ , where  $g_A$  is the axial coupling constant. The relevant components of the currents in this reaction are  $V^0$  and  $A^i$  ( $i = 1, 2, 3$ ). We take these operators as a sum of a single-nucleon operators as

$$V^0 = \sum_{a=1}^4 \tau_a^- e^{i\mathbf{q} \cdot \mathbf{r}_a}, \quad A^i = \sum_{a=1}^4 \tau_a^- \sigma_a^i e^{i\mathbf{q} \cdot \mathbf{r}_a}, \quad (10)$$

where  $\mathbf{q}$  is the momentum carried by the current,  $\mathbf{r}_a$  is the spatial coordinate of the  $a$ -th nucleon ( $a \in \{1, 2, 3, 4\}$ ), and  $\tau_a^-$  and  $\sigma_a^i$  denote the isospin ladder operator and the spin operator of the  $a$ -th nucleon, respectively. Each component leads to a part of hadronic matrix element:

$$\begin{aligned} \langle \text{tn} | V^0 | {}^4\text{He} \rangle &= \sqrt{2} \mathcal{M}_{\text{tn}}, \\ \langle \text{tn} | g_A A^+ | {}^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{tn}}, \\ \langle \text{tn} | g_A A^- | {}^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{tn}}, \\ \langle \text{tn} | g_A A^3 | {}^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{tn}}, \end{aligned} \quad (11)$$

where  $A^\pm = (A^1 \pm iA^2)/\sqrt{2}$ . Given the relevant wave functions of a  ${}^4\text{He}$  nucleus, a triton, and a neutron in Appendix, we obtain the hadronic matrix element as

$$\begin{aligned} \mathcal{M}_{\text{tn}} &= \left( \frac{128\pi}{3} \frac{a_{\text{He}} a_t^2}{(a_{\text{He}} + a_t)^4} \right)^{3/4} \\ &\quad \times \left\{ \exp\left[-\frac{\mathbf{q}_t^2}{3a_{\text{He}}}\right] - \exp\left[-\frac{\mathbf{q}_n^2}{3a_{\text{He}}} - \frac{(\mathbf{q}_t + \mathbf{q}_n)^2}{6(a_{\text{He}} + a_t)}\right] \right\}. \end{aligned} \quad (12)$$

Here  $\mathbf{q}_t$  and  $\mathbf{q}_n$  are three-momenta of the triton and the neutron, respectively, and  $a_{\text{He}}$  and  $a_t$  are related to the mean square matter radius  $R_{\text{mat}}$  by

$$a_{\text{He}} = \frac{9}{16} \frac{1}{(R_{\text{mat}})_{\text{He}}^2}, \quad a_t = \frac{1}{2} \frac{1}{(R_{\text{mat}})_t^2}. \quad (13)$$

TABLE I: Input values of the matter radius  $R_{\text{mat}}$  for d, t, and  ${}^4\text{He}$ , the magnetic radius  $R_{\text{mag}}$  for p and n, nucleus mass  $m_X$ , excess energy  $\Delta_X$  for the nucleus  $X$ , and each reference.

nucleus	$R_{\text{mat(mag)}} \text{ [fm]}/[\text{GeV}^{-1}]$	$m_X \text{ [GeV]}$	$\Delta_X \text{ [GeV]}$
p	0.876 / 4.439 [53]	0.9383 [58]	$6.778 \times 10^{-3}$ [59]
n	0.873 / 4.424 [54]	0.9396 [58]	$8.071 \times 10^{-3}$ [59]
d	1.966 / 9.962 [55]	1.876 [59]	$1.314 \times 10^{-2}$ [59]
t	1.928 / 9.770 [56]	2.809 [59]	$1.495 \times 10^{-2}$ [59]
${}^4\text{He}$	1.49 / 7.55 [57]	3.728 [59]	$2.425 \times 10^{-3}$ [59]

We list in Table I input values of the matter radius for the numerical calculation in this article.

The remaining part is straightforwardly calculated to be

$$\begin{aligned} |\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 &= |\langle \tilde{\chi}_1^0 \nu_\tau | j_z | \tilde{\tau} \rangle|^2 = 4G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2}, \\ |\langle \tilde{\chi}_1^0 \nu_\tau | j_\pm | \tilde{\tau} \rangle|^2 &= 4G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2} \left( 1 \mp \frac{p_\nu^z}{E_\nu} \right), \end{aligned} \quad (14)$$

where  $E_\nu$  and  $p_\nu^z$  are the energy and the  $z$ -component of the momentum of the tau neutrino, respectively. We assumed that the stau and the neutralino are non-relativistic. This equation includes not only all the couplings such as  $G_F$ ,  $g_L$ , and  $g_R$ , but also the effect of the virtual tau propagation in the Fig. 1. Note here that  $g_L$  coupling does not contribute. This is because the virtual tau ought to be left-handed at the weak current, and it flips its chirality during the propagation since the transferred momentum is much less than its mass.

Combining hadronic part with the other part, we obtain the squared amplitude as

$$\begin{aligned} |\mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{tn})|^2 &= \frac{8m_{\tilde{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} (1 + 3g_A^2) \mathcal{M}_{\text{tn}}^2 E_\nu. \end{aligned} \quad (15)$$

Integrating on the phase space of the final states, we obtain the cross section as

$$\begin{aligned} \sigma v_{\text{tn}} &= \frac{8}{\pi^2} \left( \frac{32}{3\pi} \right)^{3/2} g^2 \tan^2 \theta_W \sin^2 \theta_\tau (1 + 3g_A^2) G_F^2 \\ &\quad \times \Delta_{\text{tn}}^4 \frac{m_t m_n}{m_{\tilde{\tau}} m_\tau^2} \frac{a_{\text{He}}^{3/2} a_t^3}{(a_{\text{He}} + a_t)^5} I_{\text{tn}}, \end{aligned} \quad (16)$$

$$\begin{aligned}
I_{tn} = & 12 \int_0^1 ds \int_0^{\sqrt{1-s^2}} dt (1-s^2-t^2)^2 st \\
& \times \left\{ \frac{1}{6} \frac{k_t k_n}{a_{\text{He}} + a_t} st \exp \left[ -\frac{2}{3} \frac{k_t^2}{a_{\text{He}}} t^2 \right] + \frac{1}{4} \exp \left[ -\frac{2}{3} \frac{k_n^2}{a_{\text{He}}} s^2 - \frac{1}{3} \frac{k_n^2 s^2 + k_t^2 t^2}{a_{\text{He}} + a_t} \right] \sinh \left[ \frac{2}{3} \frac{k_t k_n}{a_{\text{He}} + a_t} st \right] \right. \\
& \left. - \exp \left[ -\frac{1}{3} \frac{k_n^2 s^2 + k_t^2 t^2}{a_{\text{He}}} - \frac{1}{6} \frac{k_n^2 s^2 + k_t^2 t^2}{a_{\text{He}} + a_t} \right] \sinh \left[ \frac{1}{3} \frac{k_t k_n}{a_{\text{He}} + a_t} st \right] \right\}.
\end{aligned} \tag{17}$$


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Here  $\Delta_{tn}$ ,  $k_t$ , and  $k_n$  are defined as

$$\begin{aligned}
\Delta_{tn} &\equiv \delta m + \Delta_{\text{He}} - \Delta_t - \Delta_n - E_b, \\
k_t &\equiv \sqrt{2m_t \Delta_{tn}}, \\
k_n &\equiv \sqrt{2m_n \Delta_{tn}},
\end{aligned} \tag{18}$$

where  $\Delta_X$  is the excess energy of the nucleus  $X$ , and  $E_b$  is the binding energy of  $(\tilde{\tau}^4\text{He})$  system.

**B.**  $(\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + \mathbf{d} + \mathbf{n} + \mathbf{n}$

The rate of another spallation process of Eq.(1b) is similarly calculated. The cross section is calculated to be

$$\sigma v_{\text{dnn}} = \frac{192}{\pi^4} g^2 \tan^2 \theta_W \sin^2 \theta_\tau G_F^2 \Delta_{\text{dnn}}^4 \frac{m_n m_d}{m_{\tilde{\tau}} m_\tau^2} \left( \frac{2a_d}{a_{\text{He}}(a_d + a_{\text{He}})^2} \right)^{3/2} I_{\text{dnn}}, \tag{19}$$

where

$$\begin{aligned}
I_{\text{dnn}} = & \int_0^1 ds \int_0^{\sqrt{1-s^2}} dt \int_0^{\sqrt{1-s^2-t^2}} du (1-s^2-t^2-u^2)^2 \\
& \times \left\{ (1 + 3g_A^2) a_{\text{He}} k_n^3 s t^2 u \exp \left[ -\frac{3k_d^2 s^2 + 4k_n^2 u^2}{4a_{\text{He}}} \right] \sinh \left[ \frac{k_n k_d}{a_{\text{He}}} s u \right] \right. \\
& - \sqrt{2} (1 + g_A^2) a_{\text{He}} k_n^3 s t u^2 \exp \left[ -\frac{3k_d^2 s^2 + 2k_n^2 t^2 + 2k_n^2 u^2}{4a_{\text{He}}} \right] \sinh \left[ \frac{1}{\sqrt{2}} \frac{k_n k_d}{a_{\text{He}}} s t \right] \\
& + 2\sqrt{2} g_A^2 (a_{\text{He}} + a_d) k_n^3 s t u^2 \exp \left[ -\frac{k_n^2 (2t^2 + u^2)}{2a_{\text{He}}} - \frac{k_d^2 s^2 + 2k_n^2 t^2}{4(a_{\text{He}} + a_d)} \right] \sinh \left[ \frac{1}{\sqrt{2}} \frac{k_d k_n}{a_{\text{He}} + a_d} s t \right] \\
& - 16\sqrt{2} g_A^2 \frac{a_{\text{He}}^2}{a_{\text{He}} + a_d} \sqrt{5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2} k_n s u \\
& \left. \times \exp(-A_1 k_d^2 s^2 - A_2 k_n^2 t^2 - A_3 k_n^2 u^2) \sinh(A_4 k_d k_n s t) \sinh(A_5 k_n^2 t u) \right\}.
\end{aligned} \tag{20}$$


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Here  $\Delta_{\text{dnn}}$ ,  $k_d$ ,  $k_n$  and  $A_i (i = 1 - 5)$  are defined as follows:

The rate is then obtained in the same manner as Eq. (6).

$$\begin{aligned}
\Delta_{\text{dnn}} &\equiv \delta m + \Delta_{\text{He}} - \Delta_d - 2\Delta_n - E_b, \\
k_n &\equiv \sqrt{2m_n \Delta_{\text{dnn}}}, \\
k_d &\equiv \sqrt{2m_d \Delta_{\text{dnn}}}, \\
A_1 &\equiv \frac{4a_{\text{He}} + 3a_d}{8a_{\text{He}}(a_{\text{He}} + a_d)}, \\
A_2 &\equiv \frac{22a_{\text{He}}^3 + 44a_{\text{He}}^2 a_d + 30a_{\text{He}} a_d^2 + 7a_d^3}{4a_{\text{He}}(a_{\text{He}} + a_d)(5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2)}, \\
A_3 &\equiv \frac{8a_{\text{He}}^2 + 9a_{\text{He}} a_d + 3a_d^2}{4a_{\text{He}}(5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2)}, \\
A_4 &\equiv \frac{1}{4a_{\text{He}}(a_{\text{He}} + a_d)} \sqrt{10a_{\text{He}}^2 + 12a_{\text{He}} a_d + 4a_d^2}, \\
A_5 &\equiv \frac{(a_{\text{He}} + a_d)^2}{2a_{\text{He}}(5a_{\text{He}}^2 + 6a_{\text{He}} a_d + 2a_d^2)}.
\end{aligned} \tag{21}$$

$$\text{C. } (\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + \mathbf{p} + \mathbf{n} + \mathbf{n} + \mathbf{n}$$

The cross section of spallation process of Eq. (1c) is calculated to be

$$\begin{aligned} \sigma v_{\text{pnnn}} &= \frac{8}{\pi^9} \left( \frac{32\pi^3}{a_{\text{He}}^3} \right)^{3/2} g^2 \tan^2 \theta_W \sin^2 \theta_\tau (1 + 3g_A^2) G_F^2 \\ &\times a_{\text{He}} \Delta_{\text{pnnn}}^7 \frac{m_N^5}{m_{\tilde{\tau}} m_\tau^2} I_{\text{pnnn}}, \end{aligned} \quad (22)$$

where

$$\begin{aligned} I_{\text{pnnn}} &= \int_0^1 ds \int_0^{\sqrt{1-s^2}} dt \int_0^{\sqrt{1-s^2-t^2}} du \int_0^{\sqrt{1-s^2-t^2-u^2}} dv (1-s^2-t^2-u^2-v^2)^2 st^2 uv^2 \\ &\times \left\{ \frac{1}{\sqrt{2}} \exp \left[ -\frac{k_N^2}{2a_{\text{He}}} (3s^2 + t^2 + 2u^2) \right] \sinh \left[ \frac{\sqrt{2}k_N^2}{a_{\text{He}}} su \right] - \exp \left[ -\frac{k_N^2}{2a_{\text{He}}} (3s^2 + t^2 + u^2 + v^2) \right] \sinh \left[ \frac{k_N^2}{a_{\text{He}}} su \right] \right\}, \end{aligned} \quad (23)$$

where  $\Delta_{\text{pnnn}}$  and  $k_N$  are defined as follows:

$$\begin{aligned} \Delta_{\text{pnnn}} &\equiv \delta m + \Delta_{\text{He}} - \Delta_{\text{p}} - 3\Delta_{\text{n}} - E_b, \\ k_N &\equiv \sqrt{2m_N \Delta_{\text{pnnn}}}. \end{aligned} \quad (24)$$

In this calculation, we assumed proton and neutron have an identical kinetic energy, and then the factor  $k_p$  and  $k_n$ , which are introduced to factorize their kinetic energies, are also identical.  $k_N$  is the identical factor, and here we took  $m_N = m_n$ .

The reaction rate is obtained in the same manner as Eq. (6).

#### D. Comparing the rate of spallation reaction with that of stau-catalyzed fusion

We compare the rate of the spallation and that of the stau-catalyzed fusion. We first note that the rate of stau-catalyzed fusion strongly depends on the temperature [51], and we fix the reference temperature to be 30keV. Staus begin to form a bound state with  $^4\text{He}$  at this temperature, which corresponds to cosmic time of  $10^3\text{s}$ . Thus the bound state is formed when the lifetime of staus is longer than  $10^3\text{s}$ .

Figure 2 shows the timescale of the spallation processes as a function of  $\delta m$ . The lifetime of free stau is plotted by a solid line. We took the reference values of  $m_{\tilde{\tau}} = 350\text{GeV}$ ,  $\sin \theta_\tau = 0.8$ , and  $\gamma_\tau = 0$ . The inverted rate of the stau-catalyzed fusion at the temperature of 30keV is also shown by the horizontal dashed line. Once a bound state is formed, as long as the phase space of spallation processes are open sufficiently that is  $\delta m \gtrsim 0.026\text{GeV}$ , those processes dominate over other processes. There  $\tilde{\tau}$  property is constrained to evade the over-production of d

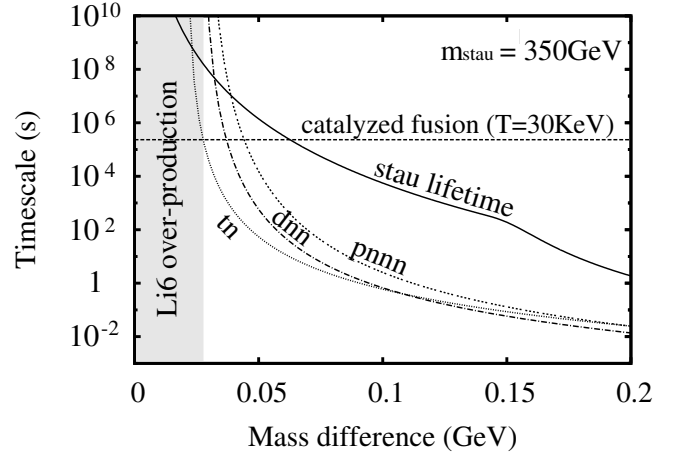


FIG. 2: Timescale of spallation processes as a function of  $\delta m$  and the stau-catalyzed fusion at the universe temperature  $T = 30\text{keV}$  [51]. The lifetime of free  $\tilde{\tau}$  (solid line) is also depicted. Here we took  $m_{\tilde{\tau}} = 350\text{GeV}$ ,  $\sin \theta_\tau = 0.8$ , and  $\gamma_\tau = 0$ .

and/or t. For  $\delta m \lesssim 0.026\text{GeV}$ , the dominant process of  $(\tilde{\tau}^4\text{He})$  is stau-catalyzed fusion, since the free  $\tilde{\tau}$  lifetime is longer than the timescale of stau-catalyzed fusion. Thus light gray region is forbidden due to the over-production of  $^6\text{Li}$ .

This interpretation of Fig.2 is not much altered by varying the parameters relevant with  $\tilde{\tau}$ . First cross sections of spallation processes are inversely proportional to  $m_{\tilde{\tau}}$ , and then the timescale of each process linearly increases as  $m_{\tilde{\tau}}$  increases. Thus, even when  $m_{\tilde{\tau}}$  is larger than  $m_{\tilde{\tau}} = 350\text{GeV}$  by up to a factor of ten, the region of  $^6\text{Li}$  over-production scarcely changes. Next we point

out that our result depend only mildly on the left-right mixing of the stau. Indeed, cross section of the  ${}^4\text{He}$  spallation is proportional to  $\sin^2 \theta_\tau$ . Its order of magnitude will not change as long as the right-handed component is significant.

### III. LIGHT ELEMENTS ABUNDANCES AND ALLOWED PARAMETER SPACE

We numerically calculate the primordial abundances of light elements including  ${}^4\text{He}$  spallation processes and  $\tilde{\tau}$  catalyzed nuclear fusion. Then we can search for allowed regions of the parameter space to fit observational light element abundances.

So far it has been reported that there is a discrepancy between the theoretical value of  ${}^7\text{Li}$  abundance predicted in the standard BBN (SBBN) and the observational one. This is called  ${}^7\text{Li}$  problem. SBBN predicts the  ${}^7\text{Li}$  to H ratio to be  $\text{Log}_{10}({}^7\text{Li}/\text{H}) = -9.35 \pm 0.06$  when we adopt a recent value of baryon to photon ratio  $\eta = (6.225 \pm 0.170) \times 10^{-10}$  (68% C.L.) reported by the WMAP satellite [60], and experimental data of the rate for the  ${}^7\text{Li}$  or  ${}^7\text{Be}$  production through  ${}^3\text{He} + {}^4\text{He} \rightarrow {}^7\text{Be} + \gamma$  [61] ( ${}^7\text{Li}$  is produced from  ${}^7\text{Be}$  by its electron capture,  ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$  at a later epoch). On the other hand, the primordial  ${}^7\text{Li}$  abundance is observed in metal-poor halo stars as absorption lines [62]. Recent observationally-inferred value of the primordial  ${}^7\text{Li}$  to hydrogen ratio is  $\text{Log}_{10}({}^7\text{Li}/\text{H}) = -9.63 \pm 0.06$  [63] for a high value, and  $\text{Log}_{10}({}^7\text{Li}/\text{H}) = -9.90 \pm 0.09$  [64] for a low value. (See also Refs. [65–67] for another values.) Therefore there is a discrepancy at more than three sigma between theoretical and observational values even when we adopt the high value of [63]. This discrepancy can be hardly attributed to the correction of the cross section of nuclear reaction [68, 69]. Even if we consider nonstandard astrophysical models such as those including diffusion effects [70, 71], it might be difficult to fit all of the data consistently [72].

In Figs. 3 and 4, we plot the allowed parameter regions which are obtained by comparing the theoretical values to observational ones for the high and low  ${}^7\text{Li}/\text{H}$ , respectively. Vertical axis is the yield value of  $\tilde{\tau}$  at the time of the formation of the bound states with nuclei,  $Y_{\tilde{\tau}} = n_{\tilde{\tau}}/s$  ( $s$  is the entropy density), and horizontal axis is the mass difference of  $\tilde{\tau}$  and  $\tilde{\chi}_1^0$ . We have adopted following another observational constraints on the light element abundances: an upper bound on the  ${}^6\text{Li}/{}^7\text{Li}$  ratio,  ${}^6\text{Li}/{}^7\text{Li} < 0.046 + 0.022$  [66], the deuteron to hydrogen ratio,  $\text{D}/\text{H} = (2.80 \pm 0.20) \times 10^{-5}$  [73], and an upper bound on the  ${}^3\text{He}$  to deuteron ratio,  ${}^3\text{He}/\text{D} < 0.87 + 0.27$  [74].

The solid line (orange line) denotes a theoretical value of the thermal relic abundance for staus [37] while keeping observationally-allowed dark matter density  $\Omega_{\text{DM}} h^2 = 0.11 \pm 0.01$  ( $2\sigma$ ) [60] as total  $\tilde{\chi}_1^0 + \tilde{\tau}$  abundance. For reference, we also plot the observationally-allowed dark matter density in the figures by a horizontal

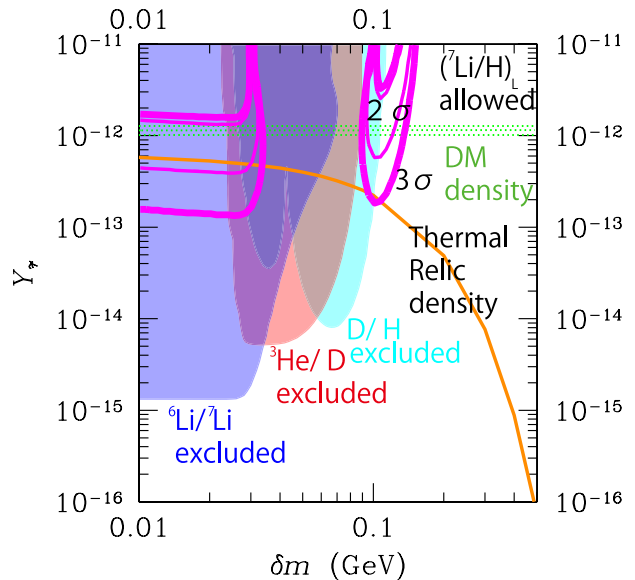


FIG. 3: Allowed regions from observational light element abundances at  $2\sigma$ . Here we have adopted the higher value of the observational  ${}^7\text{Li}/\text{H}$  in [63] denoted by  $({}^7\text{Li}/\text{H})_{\text{H}}$ , and have plotted both the  $2\sigma$  (thin line) and  $3\sigma$  (thick line) only for  ${}^7\text{Li}/\text{H}$ . The horizontal band means the observationally-allowed dark matter density. We have adopted  $m_{\tilde{\tau}} = 350$  GeV,  $\sin \theta_\tau = 0.8$ , and  $\gamma_\tau = 0$ , respectively.

band.

At around  $\delta m \sim 0.1$  GeV, we find that  ${}^7\text{Li}/\text{H}$  can be fitted to the observational value without conflicting with the other light element abundances.<sup>1</sup> As shown in Fig. 3, it should be impressive that the relic density is consistent with the allowed region at  $Y_{\tilde{\tau}} = 2 \times 10^{-13}$  at  $3\sigma$  in case of the high value of  ${}^7\text{Li}/\text{H}$  in [63].

### IV. SUMMARY

We calculated primordial abundances of all of light elements involving the helium-4 spallation processes, the catalyzed fusion, and the internal conversion processes. Newly included in the present work is the spallation of the  ${}^4\text{He}$  in the stau- ${}^4\text{He}$  bound state given in Eq.(1). This process is only present in the model which predicts the long-lived charged particles due to the phase space suppression with the weakly interacting daughter particle.

We calculated the rate of the helium-4 spallation processes analytically, and compared it with that of catalyzed fusion. We found that the spallation of  ${}^4\text{He}$  nuclei dominate over the catalyzed fusion as long as the phase

<sup>1</sup> See also [26, 39, 42, 47, 75] for another mechanisms to reduce  ${}^7\text{Li}/\text{H}$ .

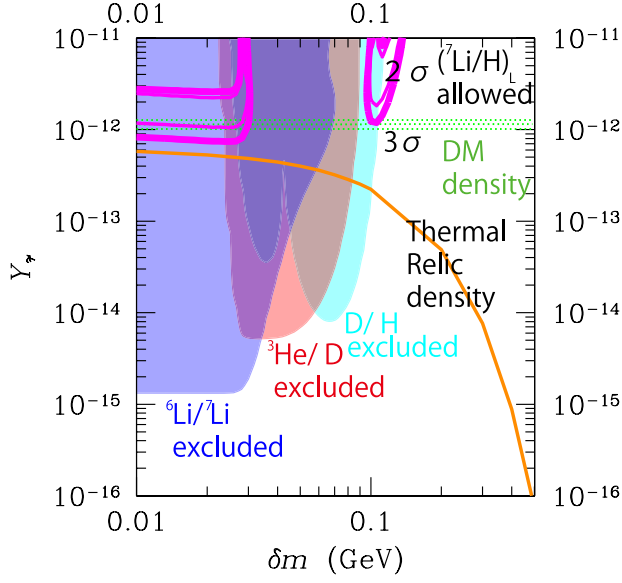


FIG. 4: Same as Fig. 3, but for the lower value of observational  ${}^7\text{Li}/\text{H}$  reported in [64], which is denoted by  $({}^7\text{Li}/\text{H})_{\text{L}}$ .

space of the spallation processes are open and hence the property of long lived stau is constrained from avoiding the overproduction of a deuteron and/or a triton. In spite of these new constraints, we found that the lithium discrepancy and the dark matter abundance can be simultaneously solved in the parameter regions presented in Fig. 3.

### Acknowledgments

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### Appendix A: Amplitude of the Spallation Reaction of ${}^4\text{He}$

The interaction relevant to our scenario is

$$\mathcal{L}_{\text{int}} = \tilde{\tau}^* \tilde{\chi}_1^0 (g_L P_L + g_R P_R) \tau + \sqrt{2} G_F (\nu_\tau \gamma^\mu P_L \tau) J_\mu + \text{H.c.}, \quad (\text{A1})$$

where  $J_\mu$  is the hadronic current. This interaction allows the spallation of the nuclei such as

$$(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + t + n, \quad (\text{A2a})$$

$$(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + d + n + n, \quad (\text{A2b})$$

$$(\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 + \nu_\tau + p + n + n + n, \quad (\text{A2c})$$

to take place. In this section, we illustrate how we calculate the amplitude of the processes of Eqs. (A2).

#### 1. ${}^4\text{He} \rightarrow t n$

The amplitude of the process (A2a) is given by

$$\mathcal{M}((\tilde{\tau} {}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau t n) = \langle t n | J^\mu | {}^4\text{He} \rangle \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle. \quad (\text{A3})$$

in which we define the leptonic matrix element by

$$\begin{aligned} \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle &= \sqrt{2} G_F \langle \tilde{\chi}_1^0 \nu_\tau | [\nu_\tau \gamma_\mu P_L \tau] [\tilde{\tau} (g_L P_R + g_R^* P_L) \tilde{\chi}_1^0 \tilde{\tau}] | \tilde{\tau} \rangle \\ & \quad (\text{A4}) \end{aligned}$$

and the hadronic matrix element by  $\langle t n | J^\mu | {}^4\text{He} \rangle$ . We separately calculate these two matrix elements.

##### a. Leptonic matrix element

The leptonic matrix element is directly calculated under the following simplifications:

- Neutralino is treated as non-relativistic particle since its mass  $m_{\tilde{\chi}_1^0}$  is much larger than the mass of nuclei.
- The momentum of a virtual tau is negligibly smaller than its mass due to the assumption that the stau and the neutralino are nearly degenerate with the mass difference of  $O(10 - 100)\text{MeV}$ .

A straightforward calculation leads to

$$\begin{aligned} |\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 &= |\langle \tilde{\chi}_1^0 \nu_\tau | j_3 | \tilde{\tau} \rangle|^2 = 4 G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2}, \\ |\langle \tilde{\chi}_1^0 \nu_\tau | j_\pm | \tilde{\tau} \rangle|^2 &= 4 G_F^2 |g_R|^2 \frac{m_{\tilde{\chi}_1^0} E_\nu}{m_\tau^2} \left( 1 \mp \frac{p_\nu^z}{E_\nu} \right), \end{aligned} \quad (\text{A5})$$

where  $E_i$ ,  $p_i$ , and  $m_i$  individually stand for energy, four-momentum and mass of particles.

##### b. Hadronic matrix element

Calculation of the hadronic matrix element requires the explicit form of the hadronic current and the wave functions of the nuclei.

We need the wave functions of initial helium, final triton, and nucleon. Building up these wave functions requires special attention to the symmetry. The wave function consists of spatial, spin, and isospin parts, and should be antisymmetric under the exchange of the two nucleons. The spin and isospin of the nucleus dictates the spin and isospin part of the wave function. We then arrange the spatial part so that the total wave function be antisymmetric under the permutation of the nucleons. We model the spatial wave functions by Gaussian functions in terms of Jacobi coordinates.

Let us make a wave function of  $^4\text{He}$  by this prescription. The spin and isospin parts of the wave function is constructed according to  $S = 0$  and  $I = 0$ , and turns out to be

$$\begin{aligned} |^4\text{He}\rangle = & \frac{1}{2\sqrt{6}} \left[ |\text{pnpn}\rangle(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle) \right. \\ & + |\text{pnmp}\rangle(-|\uparrow\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle) \\ & + |\text{nppn}\rangle(-|\uparrow\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle) \quad (\text{A6}) \\ & + |\text{npnp}\rangle(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle) \\ & + |\text{ppnn}\rangle(-|\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle) \\ & \left. + |\text{nnpp}\rangle(-|\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle) \right]. \end{aligned}$$

The above wave function is antisymmetric under the exchange of two particles. Thus the spatial part ought to be symmetric, and is constructed as

$$\begin{aligned} \psi_{\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4) = & \left( 2 \frac{a_{\text{He}}^3}{\pi^3} \right)^{3/4} \\ & \times \exp \left\{ -a_{\text{He}} \left[ \mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 + \mathbf{r}_4^2 \right. \right. \\ & \left. \left. - \frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)^2 \right] \right\} \quad (\text{A7}) \end{aligned}$$

where  $a_{\text{He}}$  parameterizes the nuclear radius and related to mean square matter radius  $R_{\text{mat}}$  by

$$a_{\text{He}} = \frac{9}{16} \frac{1}{(R_{\text{mat}})_{\text{He}}^2}. \quad (\text{A8})$$

We applied the non-relativistic normalization for the nuclear wave function, which is

$$\begin{aligned} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_3 d^3\mathbf{r}_4 |\psi_{\text{He}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4)|^2 \\ \times \delta^3\left(\frac{1}{4}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4)\right) = 1. \quad (\text{A9}) \end{aligned}$$

The wave function (A7) is independent of the coordinate of the center of mass since the initial  $^4\text{He}$  is taken to be stationary. The complete wave function of  $^4\text{He}$  nucleus is a direct product of Eqs. (A6) and (A7).

The wave functions of other nuclei are similarly obtained. For the triton, the spin and isospin part of the

wave function is formed according to  $S = 1/2$ ,  $I = 1/2$ , and  $I_z = -1/2$ ; it is given by

$$\begin{aligned} |t \uparrow\rangle = & \frac{1}{\sqrt{6}} \left[ |\text{pnn}\rangle(|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle) \right. \\ & - |\text{npn}\rangle(|\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\rangle) \quad (\text{A10a}) \\ & \left. + |\text{nnp}\rangle(|\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle) \right], \end{aligned}$$

$$\begin{aligned} |t \downarrow\rangle = & \frac{1}{\sqrt{6}} \left[ |\text{pnn}\rangle(|\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle) \right. \\ & - |\text{npn}\rangle(|\uparrow\downarrow\downarrow\rangle - |\downarrow\downarrow\uparrow\rangle) \quad (\text{A10b}) \\ & \left. + |\text{nnp}\rangle(|\uparrow\downarrow\downarrow\rangle - |\downarrow\uparrow\downarrow\rangle) \right]. \end{aligned}$$

Its spatial wave function is

$$\begin{aligned} \psi_t(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \\ = & \left( \frac{4}{3} \frac{a_t^2}{\pi^2} \right)^{3/4} \exp \left( i\mathbf{q}_t \cdot \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3) \right) \\ & \times \exp \left\{ -a_t \left[ \mathbf{r}_1^2 + \mathbf{r}_2^2 + \mathbf{r}_3^2 - \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)^2 \right] \right\}, \quad (\text{A11}) \end{aligned}$$

where  $\mathbf{q}_t$  is the center-of-mass momentum of the triton and  $a_t$  is related to the mean square matter radius as

$$a_t = \frac{1}{2} \frac{1}{(R_{\text{mat}})_t^2}. \quad (\text{A12})$$

The wave function of the final neutron is simply taken to be

$$\psi_n(\mathbf{r}) = e^{i\mathbf{q}_n \cdot \mathbf{r}} \quad (\text{A13})$$

with trivial spin and isospin parts.

Using these wave functions, we can calculate hadronic matrix elements of Eq. (A3) as

$$\begin{aligned} \langle \text{tn} | J^0 | ^4\text{He} \rangle &= \sqrt{2} \mathcal{M}_{\text{tn}}, \\ \langle \text{tn} | J^+ | ^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{tn}}, \\ \langle \text{tn} | J^- | ^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{tn}}, \\ \langle \text{tn} | J^3 | ^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{tn}}, \end{aligned} \quad (\text{A14})$$

where  $\mathcal{M}_{\text{tn}}$  is defined as

$$\begin{aligned} \mathcal{M}_{\text{tn}} = & \left( \frac{128\pi}{3} \frac{a_{\text{He}} a_t^2}{(a_{\text{He}} + a_t)^4} \right)^{3/4} \\ & \times \left\{ \exp \left[ -\frac{\mathbf{q}_t^2}{3a_{\text{He}}} \right] - \exp \left[ -\frac{\mathbf{q}_n^2}{3a_{\text{He}}} - \frac{(\mathbf{q}_t + \mathbf{q}_n)^2}{6(a_{\text{He}} + a_t)} \right] \right\}. \quad (\text{A15}) \end{aligned}$$

### c. Amplitude

Combining the hadronic matrix elements with the leptonic matrix elements, we obtain the square amplitude



of Eq. (A3) as

$$\begin{aligned}
& |\mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{tn})|^2 \\
&= |\langle \text{tn} | J^0 | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 \\
&+ |\langle \text{tn} | J^- | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_+ | \tilde{\tau} \rangle|^2 \\
&+ |\langle \text{tn} | J^+ | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_- | \tilde{\tau} \rangle|^2 \\
&+ |\langle \text{tn} | J^3 | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_3 | \tilde{\tau} \rangle|^2 \\
&= \frac{8m_{\tilde{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} (1 + 3g_A^2) \mathcal{M}_{\text{tn}}^2 E_\nu
\end{aligned} \tag{A16}$$

## 2. $^4\text{He} \rightarrow \text{dnn}$

The calculation presented in the previous section is applicable to another spallation process of Eq. (A2b). The amplitude we need is

$$\begin{aligned}
& \mathcal{M}(^4\text{He} \rightarrow \text{dnn}) \\
&= \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle \langle \text{dnn} | J^\mu | ^4\text{He} \rangle.
\end{aligned} \tag{A17}$$

The leptonic matrix element is same as in Eq. (A3) and is already calculated in Eq. (A5), while the hadronic matrix element requires a calculation anew.

### a. Hadronic matrix element

We need the wave function of the final states, which include a deuteron and two neutrons. The spin and isospin of a deuteron is  $S = 1$  and  $I = 0$ , and the corresponding wave functions are

$$|d, +1\rangle = \frac{1}{\sqrt{2}}(|p \uparrow\rangle |n \uparrow\rangle - |n \uparrow\rangle |p \uparrow\rangle), \tag{A18}$$

$$\begin{aligned}
|d, 0\rangle &= \frac{1}{2}(|p \uparrow\rangle |n \downarrow\rangle - |n \uparrow\rangle |p \downarrow\rangle \\
&\quad + |p \downarrow\rangle |n \uparrow\rangle - |n \downarrow\rangle |p \uparrow\rangle),
\end{aligned} \tag{A19}$$

$$|d, -1\rangle = \frac{1}{\sqrt{2}}(|p \downarrow\rangle |n \downarrow\rangle - |n \downarrow\rangle |p \downarrow\rangle). \tag{A20}$$

The spatial part of the wave function is given by

$$\begin{aligned}
\psi_d(\mathbf{r}_1, \mathbf{r}_2) &= \left(\frac{a_d}{\pi}\right)^{3/4} \\
&\times \exp\left(i\mathbf{q}_d \cdot \frac{\mathbf{r}_1 + \mathbf{r}_2}{2}\right) \exp\left[-\frac{1}{2}a_d(\mathbf{r}_1 - \mathbf{r}_2)^2\right],
\end{aligned} \tag{A21}$$

where  $\mathbf{q}_d$  is the center-of-mass momentum and  $a_d$  is related to the mean square matter radius as

$$a_d = \frac{3}{8} \frac{1}{(R_{\text{mat}})_d^2}. \tag{A22}$$

The spin of two neutrons can be  $S = 0$  and  $S = 1$ . For each case, spin and isospin parts of the wave functions are

$$|n_0\rangle = \frac{1}{\sqrt{2}}(|n \uparrow\rangle |n \downarrow\rangle - |n \downarrow\rangle |n \uparrow\rangle), \tag{A23}$$

$$|n_1, +1\rangle = |n \uparrow\rangle |n \uparrow\rangle, \tag{A24}$$

$$|n_1, 0\rangle = \frac{1}{\sqrt{2}}(|n \uparrow\rangle |n \downarrow\rangle + |n \downarrow\rangle |n \uparrow\rangle), \tag{A25}$$

$$|n_1, -1\rangle = |n \downarrow\rangle |n \downarrow\rangle, \tag{A26}$$

where  $|n_i\rangle$  expresses spin and isospin part of wave function of  $S = i$ . Since the spin and isospin part of  $S = 1$  are symmetric under the exchange of two particles, the spatial part of the wave function ought to be antisymmetric. The spin and isospin part of  $S = 0$ , on the other hand, is antisymmetric under the exchange of two particles, then the spatial part of the wave function ought to be symmetric. Therefore the spatial parts of each wave function are given by

$$\begin{aligned}
\psi_{n0}(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\sqrt{2}}\{\exp[i(\mathbf{q}_{n1} \cdot \mathbf{r}_1 + \mathbf{q}_{n2} \cdot \mathbf{r}_2)] \\
&\quad + \exp[i(\mathbf{q}_{n2} \cdot \mathbf{r}_1 + \mathbf{q}_{n1} \cdot \mathbf{r}_2)]\},
\end{aligned} \tag{A27}$$

$$\begin{aligned}
\psi_{n1}(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\sqrt{2}}\{\exp[i(\mathbf{q}_{n1} \cdot \mathbf{r}_1 + \mathbf{q}_{n2} \cdot \mathbf{r}_2)] \\
&\quad - \exp[i(\mathbf{q}_{n2} \cdot \mathbf{r}_1 + \mathbf{q}_{n1} \cdot \mathbf{r}_2)]\}.
\end{aligned} \tag{A28}$$

Using these wave function, we can calculate Hadronic matrix elements of Eq. (A17) as

$$\begin{aligned}
\langle \text{dn}_1 | J^0 | ^4\text{He} \rangle &= \sqrt{3} \mathcal{M}_{\text{dnn}}, \\
\langle \text{dn}_1 | J^+ | ^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{dnn}}, \\
\langle \text{dn}_1 | J^- | ^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{dnn}}, \\
\langle \text{dn}_1 | J^3 | ^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{dnn}}, \\
\langle \text{dn}_0 | J^+ | ^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}'_{\text{dnn}}, \\
\langle \text{dn}_0 | J^- | ^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}'_{\text{dnn}}, \\
\langle \text{dn}_0 | J^3 | ^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}'_{\text{dnn}},
\end{aligned} \tag{A29}$$

where  $\mathcal{M}_{\text{dnn}}$  and  $\mathcal{M}'_{\text{dnn}}$  are defined as

$$\begin{aligned}\mathcal{M}_{\text{dnn}} &= \left( \frac{32\pi^2 a_d}{a_{\text{He}}(a_{\text{He}} + a_d)^2} \right)^{\frac{3}{4}} \left[ \exp \left( -\frac{4\mathbf{q}_{n2}^2 + 4\mathbf{q}_{n2} \cdot \mathbf{q}_d + 3\mathbf{q}_d^2}{8a_{\text{He}}} \right) - \exp \left( -\frac{4\mathbf{q}_{n1}^2 + 4\mathbf{q}_{n1} \cdot \mathbf{q}_d + 3\mathbf{q}_d^2}{8a_{\text{He}}} \right) \right], \\ \mathcal{M}'_{\text{dnn}} &= \frac{1}{\sqrt{2}} \left( \frac{32\pi^2 a_d}{a_{\text{He}}(a_{\text{He}} + a_d)^2} \right)^{\frac{3}{4}} \left[ \exp \left( -\frac{4\mathbf{q}_{n2}^2 + 4\mathbf{q}_{n2} \cdot \mathbf{q}_d + 3\mathbf{q}_d^2}{8a_{\text{He}}} \right) + \exp \left( -\frac{4\mathbf{q}_{n1}^2 + 4\mathbf{q}_{n1} \cdot \mathbf{q}_d + 3\mathbf{q}_d^2}{8a_{\text{He}}} \right) \right. \\ &\quad \left. - 2 \exp \left( -\frac{3\mathbf{q}_{n1}^2 + 2\mathbf{q}_{n1} \cdot \mathbf{q}_{n2} + 3\mathbf{q}_{n2}^2}{8a_{\text{He}}} - \frac{1}{8} \frac{(\mathbf{q}_d + \mathbf{q}_{n1} + \mathbf{q}_{n2})^2}{a_{\text{He}} + a_d} \right) \right].\end{aligned}\quad (\text{A30})$$

### b. Amplitude

Combining the hadronic matrix elements with the leptonic elements, we obtain the square amplitude of Eq. (A17) as

$$\begin{aligned}|\mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{dnn})|^2 &= |\langle \text{dnn} | J^0 | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 \\ &+ |\langle \text{dnn} | J^- | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_+ | \tilde{\tau} \rangle|^2 \\ &+ |\langle \text{dnn} | J^+ | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_- | \tilde{\tau} \rangle|^2 \\ &+ |\langle \text{dnn} | J^3 | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_3 | \tilde{\tau} \rangle|^2 \\ &= \frac{12m_{\tilde{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} ((1 + 2g_A^2) \mathcal{M}_{\text{dnn}}^2 + 2g_A^2 \mathcal{M}_{\text{dnn}}'^2) E_\nu.\end{aligned}\quad (\text{A31})$$

### 3. $^4\text{He} \rightarrow \text{pnnn}$

The matrix element for the  $^4\text{He} \rightarrow \text{pnnn}$  process is

$$\begin{aligned}\mathcal{M}(^4\text{He} \rightarrow \text{pnnn}) &= \langle \tilde{\chi}_1^0 \nu_\tau | j_\mu | \tilde{\tau} \rangle \langle \text{pnnn} | J^\mu | ^4\text{He} \rangle.\end{aligned}\quad (\text{A32})$$

The calculation is also performed with an identical step as that of other  $^4\text{He}$  spallation processes.

#### a. Hadronic matrix element

The final state of the process is a system composed of a proton and three neutrons, and two types of the systems could be brought; (1)  $S = 0$  and  $S_z = 0$  via vector current (2)  $S = 1$  and  $S_z = \{-1, 0, +1\}$  via axial vector current.

The spin and isospin part of the system for  $S = 0$  and  $S_z = 0$  is given by

$$\begin{aligned}|\text{pnnn}(S = 0, S_z = 0)\rangle &= \frac{1}{4\sqrt{3}} \epsilon_{ijkl} \\ &\times \left[ |n \uparrow\rangle_i |n \uparrow\rangle_j |n \downarrow\rangle_k |p \downarrow\rangle_l + |n \downarrow\rangle_i |n \downarrow\rangle_j |n \uparrow\rangle_k |p \uparrow\rangle_l \right],\end{aligned}\quad (\text{A33})$$

where we set  $\epsilon_{ijkl} = +1$ . Here indices  $i, j, k, l$  are corresponding to spacial coordinates, i.e., spacial part of the wave function for  $|n \uparrow\rangle_i |n \uparrow\rangle_j |n \downarrow\rangle_k |p \downarrow\rangle_l$  is

$$\begin{aligned}\psi(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k, \mathbf{r}_l) &\equiv \exp \left[ i(\mathbf{q}_{n1} \cdot \mathbf{r}_i + \mathbf{q}_{n2} \cdot \mathbf{r}_j + \mathbf{q}_{n3} \cdot \mathbf{r}_k + \mathbf{q}_p \cdot \mathbf{r}_l) \right].\end{aligned}\quad (\text{A34})$$

Here each spacial part of proton and neutron are simply taken to be plane wave form. Similarly, the spin and isospin parts of the system for  $S = 1$  and  $S_z = \{-1, 0, +1\}$  are given by

$$\begin{aligned}|\text{pnnn}(S = 1, S_z = +1)\rangle &= \frac{1}{2\sqrt{6}} \epsilon_{ijkl} |n \uparrow\rangle_i |n \uparrow\rangle_j |n \downarrow\rangle_k |p \uparrow\rangle_l\end{aligned}\quad (\text{A35})$$

$$\begin{aligned}|\text{pnnn}(S = 1, S_z = 0)\rangle &= \frac{1}{4\sqrt{3}} \epsilon_{ijkl} \\ &\times \left[ |n \uparrow\rangle_i |n \uparrow\rangle_j |n \downarrow\rangle_k |p \downarrow\rangle_l - |n \downarrow\rangle_i |n \downarrow\rangle_j |n \uparrow\rangle_k |p \uparrow\rangle_l \right]\end{aligned}\quad (\text{A36})$$

$$\begin{aligned}|\text{pnnn}(S = 1, S_z = -1)\rangle &= \frac{1}{2\sqrt{6}} \epsilon_{ijkl} |n \downarrow\rangle_i |n \downarrow\rangle_j |n \uparrow\rangle_k |p \downarrow\rangle_l\end{aligned}\quad (\text{A37})$$

Spatial parts of them are same as the system for  $S = 0$  and  $S_z = 0$ .

The hadronic matrix element in (A32) is calculated with built wave functions and explicit form of each current as follows,

$$\begin{aligned}\langle \text{pnnn} | J^0 | ^4\text{He} \rangle &= \sqrt{2} \mathcal{M}_{\text{pnnn}}, \\ \langle \text{pnnn} | J^+ | ^4\text{He} \rangle &= -\sqrt{2} g_A \mathcal{M}_{\text{pnnn}}, \\ \langle \text{pnnn} | J^- | ^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{pnnn}}, \\ \langle \text{pnnn} | J^3 | ^4\text{He} \rangle &= \sqrt{2} g_A \mathcal{M}_{\text{pnnn}},\end{aligned}\quad (\text{A38})$$

where  $\mathcal{M}_{\text{pnnn}}$  is define as follow:

$$\mathcal{M}_{\text{pnnn}} = \left( \frac{32\pi^3}{a_{\text{He}}^3} \right)^{3/4} \left\{ \exp \left[ -\frac{1}{2a_{\text{He}}} (\mathbf{q}_{n2}^2 + \mathbf{q}_{n3}^2 + \mathbf{q}_p^2 + \mathbf{q}_{n2} \cdot \mathbf{q}_{n3} + \mathbf{q}_{n2} \cdot \mathbf{q}_p + \mathbf{q}_{n3} \cdot \mathbf{q}_p) \right] \right. \\ \left. - \exp \left[ -\frac{1}{2a_{\text{He}}} (\mathbf{q}_{n1}^2 + \mathbf{q}_{n3}^2 + \mathbf{q}_p^2 + \mathbf{q}_{n1} \cdot \mathbf{q}_{n3} + \mathbf{q}_{n1} \cdot \mathbf{q}_p + \mathbf{q}_{n3} \cdot \mathbf{q}_p) \right] \right\} \quad (\text{A39})$$

### b. Amplitude

Combining the hadronic matrix elements with the leptonic matrix elements, we obtain the square amplitude of Eq. (A32) as

$$|\mathcal{M}((\tilde{\tau}^4\text{He}) \rightarrow \tilde{\chi}_1^0 \nu_\tau \text{pnnn})|^2 \\ = |\langle \text{pnnn} | J^0 | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_0 | \tilde{\tau} \rangle|^2 \\ + |\langle \text{pnnn} | J^- | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_- | \tilde{\tau} \rangle|^2 \\ + |\langle \text{pnnn} | J^+ | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_+ | \tilde{\tau} \rangle|^2 \\ + |\langle \text{pnnn} | J^3 | ^4\text{He} \rangle|^2 |\langle \tilde{\chi}_1^0 \nu_\tau | j_3 | \tilde{\tau} \rangle|^2 \quad (\text{A40}) \\ = \frac{8m_{\tilde{\chi}_1^0} G_F^2 |g_R|^2}{m_\tau^2} (1 + 3g_A^2) \mathcal{M}_{\text{pnnn}}^2 E_\nu.$$

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- [1] M. Drees and X. Tata, Phys. Lett. B **252** (1990) 695.
  - [2] J. L. Feng and T. Moroi, Phys. Rev. D **58** (1998) 035001 [arXiv:hep-ph/9712499].
  - [3] S. P. Martin and J. D. Wells, Phys. Rev. D **59** (1999) 035008 [arXiv:hep-ph/9805289].
  - [4] K. Hamaguchi, Y. Kuno, T. Nakaya and M. M. Nojiri, Phys. Rev. D **70** (2004) 115007 [arXiv:hep-ph/0409248].
  - [5] J. L. Feng and B. T. Smith, Phys. Rev. D **71** (2005) 015004 [Erratum-ibid. D **71** (2005) 019904] [arXiv:hep-ph/0409278].
  - [6] A. Ibarra and S. Roy, JHEP **0705** (2007) 059 [arXiv:hep-ph/0606116].
  - [7] K. Hamaguchi, M. M. Nojiri and A. de Roeck, JHEP **0703** (2007) 046 [arXiv:hep-ph/0612060].
  - [8] J. L. Feng, C. G. Lester, Y. Nir and Y. Shadmi, Phys. Rev. D **77** (2008) 076002 [arXiv:0712.0674 [hep-ph]].
  - [9] S. Kaneko, J. Sato, T. Shimomura, O. Vives and M. Yamanaka, Phys. Rev. D **78** (2008) 116013 [arXiv:0811.0703 [hep-ph]].
  - [10] S. Asai, K. Hamaguchi and S. Shirai, Phys. Rev. Lett. **103** (2009) 141803 [arXiv:0902.3754 [hep-ph]].
  - [11] T. Ito, R. Kitano and T. Moroi, JHEP **1004**, 017 (2010) [arXiv:0910.5853 [hep-ph]].
  - [12] T. Ito and T. Moroi, Phys. Lett. B **694** (2011) 349 [arXiv:1007.3060 [hep-ph]].
  - [13] S. Kaneko, H. Saito, J. Sato, T. Shimomura, O. Vives and M. Yamanaka, arXiv:1102.1794 [hep-ph].
  - [14] S. Asai, Y. Azuma, M. Endo, K. Hamaguchi and S. Iwamoto, arXiv:1103.1881 [hep-ph].
  - [15] M. Ahlers, J. Kersten and A. Ringwald, JCAP **0607** (2006) 005 [arXiv:hep-ph/0604188].
  - [16] S. Ando, J. F. Beacom, S. Profumo and D. Rainwater, JCAP **0804**, 029 (2008) [arXiv:0711.2908 [hep-ph]].
  - [17] B. Canadas, D. G. Cerdeno, C. Munoz and S. Panda, JCAP **0904** (2009) 028 [arXiv:0812.1067 [hep-ph]].
  - [18] K. Sigurdson and M. Kamionkowski, Phys. Rev. Lett. **92** (2004) 171302 [arXiv:astro-ph/0311486].
  - [19] J. Hisano, K. T. Inoue and T. Takahashi, Phys. Lett. B **643** (2006) 141 [arXiv:hep-ph/0608126].
  - [20] F. Borzumati, T. Bringmann and P. Ullio, Phys. Rev. D **77** (2008) 063514 [arXiv:hep-ph/0701007].
  - [21] K. Kohri and T. Takahashi, Phys. Lett. B **682** (2010) 337 [arXiv:0909.4610 [hep-ph]].
  - [22] M. Pospelov, Phys. Rev. Lett. **98**, 231301 (2007) [arXiv:hep-ph/0605215].
  - [23] K. Kohri and F. Takayama, Phys. Rev. D **76**, 063507 (2007) [arXiv:hep-ph/0605243].
  - [24] M. Kaplinghat and A. Rajaraman, Phys. Rev. D **74**, 103004 (2006) [arXiv:astro-ph/0606209].
  - [25] R. H. Cyburt, J. R. Ellis, B. D. Fields, K. A. Olive and V. C. Spanos, JCAP **0611**, 014 (2006) [arXiv:astro-ph/0608562].
  - [26] C. Bird, K. Koopmans and M. Pospelov, Phys. Rev. D **78**, 083010 (2008) [arXiv:hep-ph/0703096].
  - [27] M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B **649**, 436 (2007) [arXiv:hep-ph/0703122].
  - [28] T. Jittoh, K. Kohri, M. Koike, J. Sato, T. Shimomura and M. Yamanaka, Phys. Rev. D **76** (2007) 125023 [arXiv:0704.2914 [hep-ph]].
  - [29] K. Jedamzik, Phys. Rev. D **77**, 063524 (2008) [arXiv:0707.2070 [astro-ph]].
  - [30] J. Pradler and F. D. Steffen, Phys. Lett. B **666**, 181 (2008) [arXiv:0710.2213 [hep-ph]].
  - [31] M. Kawasaki, K. Kohri, T. Moroi and A. Yotsuyanagi, Phys. Rev. D **78**, 065011 (2008) [arXiv:0804.3745 [hep-ph]].

- [32] T. Jittoh, K. Kohri, M. Koike, J. Sato, T. Shimomura and M. Yamanaka, Phys. Rev. D **78** (2008) 055007 [arXiv:0805.3389 [hep-ph]].
- [33] M. Pospelov, J. Pradler and F. D. Steffen, JCAP **0811**, 020 (2008) [arXiv:0807.4287 [hep-ph]].
- [34] M. Kamimura, Y. Kino and E. Hiyama, Prog. Theor. Phys. **121**, 1059 (2009) [arXiv:0809.4772 [nucl-th]].
- [35] S. Bailly, K. Jedamzik and G. Moulataka, Phys. Rev. D **80**, 063509 (2009) [arXiv:0812.0788 [hep-ph]].
- [36] S. Bailly, K. Y. Choi, K. Jedamzik and L. Roszkowski, JHEP **0905**, 103 (2009) [arXiv:0903.3974 [hep-ph]].
- [37] T. Jittoh, K. Kohri, M. Koike, J. Sato, T. Shimomura and M. Yamanaka, Phys. Rev. D **82** (2010) 115030 [arXiv:1001.1217 [hep-ph]].
- [38] M. Kusakabe, T. Kajino, T. Yoshida and G. J. Mathews, Phys. Rev. D **81**, 083521 (2010) [arXiv:1001.1410 [astro-ph.CO]].
- [39] M. Kawasaki and M. Kusakabe, Phys. Rev. D **83**, 055011 (2011) [arXiv:1012.0435 [hep-ph]].
- [40] M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B **625**, 7 (2005) [arXiv:astro-ph/0402490].
- [41] M. Kawasaki, K. Kohri and T. Moroi, Phys. Rev. D **71**, 083502 (2005) [arXiv:astro-ph/0408426].
- [42] K. Jedamzik, Phys. Rev. D **70**, 063524 (2004) [arXiv:astro-ph/0402344].
- [43] D. Cumberbatch, K. Ichikawa, M. Kawasaki, K. Kohri, J. Silk and G. D. Starkman, Phys. Rev. D **76**, 123005 (2007) [arXiv:0708.0095 [astro-ph]].
- [44] M. Kusakabe, T. Kajino, T. Yoshida, T. Shima, Y. Nagai and T. Kii, Phys. Rev. D **79**, 123513 (2009) [arXiv:0806.4040 [astro-ph]].
- [45] K. Kohri and Y. Santoso, Phys. Rev. D **79**, 043514 (2009) [arXiv:0811.1119 [hep-ph]].
- [46] R. H. Cyburt, J. Ellis, B. D. Fields, F. Luo, K. A. Olive and V. C. Spanos, JCAP **0910**, 021 (2009) [arXiv:0907.5003 [astro-ph.CO]].
- [47] M. Pospelov and J. Pradler, Phys. Rev. D **82**, 103514 (2010) [arXiv:1006.4172 [hep-ph]].
- [48] K. Griest and D. Seckel, Phys. Rev. D **43** (1991) 3191.
- [49] S. Profumo, K. Sigurdson, P. Ullio and M. Kamionkowski, Phys. Rev. D **71** (2005) 023518 [arXiv:astro-ph/0410714].
- [50] T. Jittoh, J. Sato, T. Shimomura and M. Yamanaka, Phys. Rev. D **73** (2006) 055009 [arXiv:hep-ph/0512197].
- [51] K. Hamaguchi, T. Hatsuda, M. Kamimura, Y. Kino and T. T. Yanagida, Phys. Lett. B **650** (2007) 268 [arXiv:hep-ph/0702274].
- [52] W. Buchmuller, L. Covi, K. Hamaguchi, A. Ibarra and T. Yanagida, JHEP **0703**, 037 (2007) [arXiv:hep-ph/0702184].
- [53] D. Borisyuk, Nucl. Phys. A **843**, 59 (2010) [arXiv:0911.4091 [hep-ph]].
- [54] G. Kubon *et al.*, Phys. Lett. B **524** (2002) 26 [arXiv:nucl-ex/0107016].
- [55] C. W. Wong, Int. J. Mod. Phys. E **3**, 821 (1994).
- [56] M. Yoshitake *et al.*, private communication.
- [57] P. Egelhof, Prog. Part. Nucl. Phys. **46**, 307-316 (2001)
- [58] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**, 633 (2008)
- [59] R. B. Firestone, *Table of Isotopes. Eighth Edition Volume I*, John Wiley and Sons Inc, New York, (1999)
- [60] E. Komatsu *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].
- [61] R. H. Cyburt, B. D. Fields and K. A. Olive, JCAP **0811** (2008) 012 [arXiv:0808.2818 [astro-ph]].
- [62] F. Spite and M. Spite, Astron. Astrophys. **115** (1982) 357.
- [63] J. Melendez and I. Ramirez, Astrophys. J. **615** (2004) L33 [arXiv:astro-ph/0409383].
- [64] P. Bonifacio *et al.*, Astron. Astrophys. **462** (2007) 851 [arXiv:astro-ph/0610245].
- [65] S. G. Ryan, T. C. Beers, K. A. Olive, B. D. Fields and J. E. Norris, Astrophys. J. **530** (2000) L57.
- [66] M. Asplund, D. L. Lambert, P. E. Nissen, F. Primas and V. V. Smith, Astrophys. J. **644** (2006) 229 [arXiv:astro-ph/0510636].
- [67] W. Aoki *et al.*, Astrophys. J. **698** (2009) 1803 [arXiv:0904.1448 [astro-ph.SR]].
- [68] R. H. Cyburt, B. D. Fields and K. A. Olive, Phys. Rev. D **69** (2004) 123519 [arXiv:astro-ph/0312629].
- [69] C. Angulo *et al.*, Astrophys. J. **630** (2005) L105 [arXiv:astro-ph/0508454].
- [70] O. Richard, G. Michaud and J. Richer, Astrophys. J. **619**, 538 (2005) [arXiv:astro-ph/0409672].
- [71] A. J. Korn *et al.*, Nature **442** (2006) 657 [arXiv:astro-ph/0608201].
- [72] K. Lind, F. Primas, C. Charbonnel, F. Grundahl and M. Asplund, arXiv:0906.2876 [astro-ph.SR].
- [73] M. Pettini, B. J. Zych, M. T. Murphy, A. Lewis and C. C. Steidel, MNRAS, **391** (2008) 1499 [arXiv:0805.0594 [astro-ph]].
- [74] J. Geiss and G. Gloeckler, Space Science Reviews **106**, 3 (2003).
- [75] O. Erken, P. Sikivie, H. Tam and Q. Yang, arXiv:1104.4507 [astro-ph.CO].